

Electron in an Ultrashort Laser Pulse

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After the classical approach to acceleration of a charged particle by δ -form impulsive force, we consider the corresponding quantum theory based on the Volkov solution of the Dirac equation. We determine the modified Compton formula for frequency of photons generated by the scattering of the δ -form laser pulse on the electron in a rest frame.

KEY WORDS: laser pulse; Volkov solution; Compton effect.

1. INTRODUCTION

The problem of interaction of elementary particles with a laser field is, at present time, one of the most prestigious problems in particle physics. It is supposed that in the future, the laser will play the same role in particle physics as the linear or circle accelerators used in today's particle laboratories.

One of the problems is acceleration of charged particles by the laser. The acceleration effectiveness of the linear or circle accelerators is limited not only by their geometrical size of them but also by the energy loss of accelerated particles, which is caused by bremsstrahlung during the acceleration. The amount of radiation follows from the Larmor formula for emission of radiation by an accelerated charged particle (Maier, 1991).

In case of laser acceleration the classical idea is that acceleration is caused by laser light as the periodic electromagnetic field. However, it is possible to show that periodic electromagnetic wave does not accelerate electrons in classical and quantum theory, because the electric and magnetic components of the light field are mutually perpendicular and it means the motion caused by the classical periodic electromagnetic field is not linear but periodic (Landau and Lifshitz, 1962). Similarly, the quantum motion of electron in such a wave firstly described by Volkov (1935) must coincide in the classical limit with the classical solution.

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The situation changes if we consider laser beam as a system of photons, and the interaction of electron with laser light is via the one-photon Compton process

$$\gamma + e \rightarrow \gamma + e, \quad (1a)$$

or via the multiphoton Compton process

$$n\gamma + e \rightarrow \gamma + e, \quad (1b)$$

where n is a natural number.

Equation (1b) is the symbolic expression of the two different physical processes. One process is the nonlinear Compton effect in which several photons are absorbed at a single point, but only single high-energy photon is emitted. The second process is interaction where electron scatters twice or more as it traverses the laser focus. In our paper the attention is devoted to the nonlinear Compton process.

It is evident that acceleration by laser can be adequately described only by quantum electrodynamics. Such viewpoint gives us the motivation to investigate theoretically the effectiveness of acceleration of charged particles by laser beam. The acceleration of charged particles by laser beam has been studied by many authors (Baranova *et al.*, 1994; Katsouleas and Dawson, 1983; Scully and Zubary, 1991; Tajima and Dawson, 1979). Many designs for such devices have been proposed. Some of these are not sufficiently developed to be readily intelligible, others seem to be fallacious, and still others are unlikely to be relevant to ultrahigh energies. Some designs were developed only to observe pressure of laser light on microparticles in liquids and gases (Ashkin, 1970, 1972).

It is necessary to say that acceleration by the Compton effect differs from the effect of light pressure, which was considered in past, for instance, by Russian physicists Lebedev and Nichols and Hull (1903) and which was also verified experimentally by these scientists. The measurement consisted in determination of force acting on the torsion pendulum. It was confirmed that the pressure is very small. Only after invention of lasers the situation changed because of the very strong intensity of the laser light which can cause the great pressure of the laser ray on the surface of the condensed matter.

Here we consider the interaction of an electron with a laser pulse. First, we consider the classical approach to acceleration of a charged particle by δ -form impulsive force. Then, we discuss, the corresponding quantum theory based on the Volkov solution of the Dirac equation. We determine the modified Compton formula for frequency of photons generated by the scattering of the δ -form laser pulse on the electron in a rest frame.

2. CLASSICAL THEORY OF INTERACTION OF PARTICLE WITH AN IMPULSIVE FORCE

We idealize the impulsive force by the dirac δ -function. Newton's second law for the interaction of a massive particle with mass m with an impulsive force $P\delta(t)$ is as follows:

$$m \frac{d^2x}{dt^2} = P\delta(t), \quad (2)$$

where P is some constant.

Using the Laplace transform on the last equation, with

$$\int_0^{\infty} e^{-st} x(t) dt = X(s), \quad (3)$$

$$\int_0^{\infty} e^{-st} \ddot{x}(t) dt = s^2 X(s) - sx(0) - \dot{x}(0), \quad (4)$$

$$\int_0^{\infty} e^{-st} \delta(t) dt = 1, \quad (5)$$

we obtain

$$ms^2 X(s) - msx(0) - m\dot{x}(0) = P. \quad (6)$$

For a particle starting from rest with $\dot{x}(0) = 0$ and $x(0) = 0$, we get

$$X(s) = \frac{P}{ms^2}, \quad (7)$$

and using the inverse Laplace transform, we obtain

$$x(t) = \frac{P}{m} t \quad (8)$$

and

$$\dot{x}(t) = \frac{P}{m}. \quad (9)$$

Let us remark that if we express δ -function by the relation $\delta(t) = \dot{\eta}(t)$, then from Eq. (2) $\dot{x}(t) = P/m$ follows immediately. The physical meaning of the quantity P can be deduced from equation $F = P\delta(t)$. After t -integration we have $\int F dt = mv = P$, where m is mass of a body and v its final velocity (with $v(0) = 0$). It means that the value of P can be determined a posteriori and then this value can be used in more complex equations than Eq. (2). Of course it is necessary to suppose that δ -form of the impulsive force is adequate approximation of the experimental situation.

In case of the harmonic oscillator with the damping force and under the influence of the general force, the Newton law is as follows:

$$m \frac{d^2 x(t)}{dt^2} + b \dot{x}(t) + kx(t) = F(t). \quad (10)$$

After application of the Laplace transform and with regard to the same initial conditions as in the preceding situation, $\dot{x}(0) = 0$, $x(0) = 0$, we get the following algebraic equation:

$$ms^2 X(s) + bsX(s) + kX(s) = F(s), \quad (11)$$

or

$$X(s) = \frac{F(s)}{m\omega_1} \frac{\omega_1}{(s + b/2m)^2 + \omega_1^2} \quad (12)$$

with $\omega_1^2 = k/m - b^2/4m^2$.

Using inverse Laplace transform denoted by symbol \mathcal{L}^{-1} applied to multiplication of functions $f_1(s)f_2(s)$,

$$\mathcal{L}^{-1}(f_1(s)f_2(s)) = \int_0^t d\tau F_1(t - \tau)F_2(\tau), \quad (13)$$

we obtain with $f_1(s) = F(s)/m\omega_1$, $f_2(s) = \omega_1/((s + b/2m)^2 + \omega_1^2)$, $F_1(t) = F(t)/m\omega_1$, $F_2(t) = \exp(-bt/2m) \sin \omega_1 t$

$$x(t) = \frac{1}{m\omega_1} \int_0^t F(t - \tau) e^{-\frac{b}{2m}\tau} \sin(\omega_1 \tau) d\tau. \quad (14)$$

For impulsive force $F(t) = P\delta(t)$ we have from the last formula

$$x(t) = \frac{P}{m\omega_1} e^{-\frac{b}{2m}t} \sin \omega_1 t. \quad (15)$$

3. CLASSICAL INTERACTION OF A CHARGED PARTICLE WITH A LASER PULSE

If we consider the δ -form electromagnetic pulse, then we can write

$$F_{\mu\nu} = a_{\mu\nu}\delta(\varphi), \quad (16)$$

where $\varphi = kx = \omega t - kx$. In order to obtain the electromagnetic impulsive force in this form, it is necessary to define the four-potential in the following form:

$$A_\mu = a_\mu \eta(\varphi), \quad (17)$$

where function η is the Heaviside unit step function defined by the relation

$$\eta(\varphi) = \begin{cases} 0, & \varphi < 0 \\ 1, & \varphi \geq 0 \end{cases}. \quad (18)$$

If we define the four-potential by Eq. (17), then the electromagnetic tensor with impulsive force is of the form

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (k_\mu a_\nu - k_\nu a_\mu)\delta(\varphi) = a_{\mu\nu}\delta(\varphi). \quad (19)$$

To find motion of an electron in δ -form electromagnetic force, following Meyer (1971), we apply his method and then replace $A_\mu(\varphi)$ by $a_\mu\eta(\varphi)$ in the final result.

The Lorentz equation with $A_\mu = a_\mu A(\varphi)$ reads

$$\frac{dp_\mu}{d\tau} = \frac{e}{m} F_{\mu\nu} p^\nu = \frac{e}{m} (k_\mu a \cdot p - a_\mu k \cdot p) A'(\varphi), \quad (20)$$

where the prime denotes derivation with regard to φ , τ is proper time, and $p_\mu = m(dx_\mu/d\tau)$. After multiplication of the last equation by k^μ , we get with regard to the Lorentz condition $0 = \partial_\mu A^\mu = a^\mu \partial_\mu A(\varphi) = k_\mu a^\mu A'$, or $k \cdot a = 0$ and $k^2 = 0$, the following equation:

$$\frac{d(k \cdot p)}{d\tau} = 0, \quad (21)$$

and it means that $k \cdot p$ is a constant of the motion and it can be defined by the initial conditions for instance at time $\tau = 0$. If we put $p_\mu(\tau = 0) = p_\mu^0$, then we can write $k \cdot p = k \cdot p^0$. At this moment we have

$$k \cdot p = \frac{mk \cdot dx}{d\tau} = m \frac{d\varphi}{d\tau}, \quad (22)$$

or

$$\frac{d\varphi}{d\tau} = \frac{k \cdot p^0}{m}. \quad (23)$$

So, using the last equation and relation $d/d\tau = (d/\varphi)d\varphi/d\tau$, we can write Eq. (20) in the form

$$\frac{dp_\mu}{d\varphi} = \frac{e}{k \cdot p^0} (k_\mu a \cdot p - a_\mu k \cdot p^0) A'(\varphi), \quad (24)$$

giving (after multiplication by a^μ)

$$\frac{d(a \cdot p)}{d\varphi} = -ea^2 A', \quad (25)$$

or

$$a \cdot p = a \cdot p^0 - ea^2 A. \quad (26)$$

Substituting the last formula into (24), we get

$$\frac{dp_\mu}{d\varphi} = -e \left(a_\mu - \frac{k_\mu a \cdot p^0}{k \cdot p^0} \right) \frac{dA}{d\varphi} - \frac{e^2 a^2}{2k \cdot p^0} \frac{d(A)^2}{d\varphi} k_\mu. \quad (27)$$

This equation can be immediately integrated to give the resulting momentum in the form

$$p_\mu = p_\mu^0 - e \left(A_\mu - \frac{A^\nu p_\nu^0 k_\mu}{k \cdot p^0} \right) - \frac{e^2 A^\nu A_\nu k_\mu}{2k \cdot p^0}. \quad (28)$$

Now, if we put into this formula the four-potential (17) of the impulsive force, then for $\varphi > 0$ when $\eta > 1$, we get

$$p_\mu = p_\mu^0 - e \left(a_\mu - \frac{a^\nu p_\nu^0 k_\mu}{k \cdot p^0} \right) - \frac{e^2 a^\nu a_\nu k_\mu}{2k \cdot p^0}. \quad (29)$$

This equation can be used to determine of the magnitude of a_μ similar to what was done in discussion of Eq. (2). It can be evidently expressed as the number of k -photons in electromagnetic momentum. For $\varphi < 0$, it is $\eta = 0$ and therefore $p_\mu = p_\mu^0$.

It is still necessary to say what is the practical realization of the δ -form potential. We know from the Fourier analysis that the Dirac δ -function can be expressed by integral in the following form:

$$\delta(\varphi) = \frac{1}{\pi} \int_0^\infty \cos(s\varphi) ds. \quad (30)$$

So, the δ -potential can be realized as the continual superposition of the harmonic waves. In case it will not be possible to realize it experimentally, we can approximate the integral formula by the summation formula as follows:

$$\delta(\varphi) \approx \frac{1}{\pi} \sum_0^\infty \cos(s\varphi). \quad (31)$$

4. VOLKOV SOLUTION OF THE DIRAC EQUATION WITH HEAVISIDE FOUR-POTENTIAL

We know that the four-potential is inbuilt in the Dirac equation and we also know that if the potential is dependent on φ , then there is explicit solution of the Dirac equation which was found by Volkov (1935) and which is called Volkov solution. The quantum mechanical problem is to find solution of the Dirac equation with the δ -form four-potential (17) and from this solution determine the quantum motion of the charged particle under this potential. Let us first remember the Volkov solution of the Dirac equation

$$(\gamma(p - eA) - m)\Psi = 0. \quad (32)$$

Volkov (1935) found the explicit solution of this equation for four-potential $A_\mu = A_\mu(\varphi)$, where $\varphi = kx$. His solution is of the form (Berestetzki *et al.*, 1989)

$$\Psi_p = R \frac{u}{\sqrt{2p_0}} e^{iS} = \left[1 + \frac{e}{2(kp)} (\gamma k)(\gamma A) \right] \frac{u}{\sqrt{2p_0}} e^{iS}, \quad (33)$$

where u is an electron bispinor of the corresponding Dirac equation

$$(\gamma p - m)u = 0. \quad (34)$$

The mathematical object S is the classical Hamilton–Jacobi function, which was determined in the form

$$S = -px - \int_0^{kx} \frac{e}{(kp)} \left[(pA) - \frac{e}{2}A^2 \right] d\varphi. \quad (35)$$

If we write Volkov wave function Ψ_p in the form (33), then for the impulsive vector potential (17) we have

$$S = -px - \left[e \frac{ap}{kp} - \frac{e^2}{2kp} a^2 \right] \varphi, \quad R = \left[1 + \frac{e}{2kp} (\gamma k)(\gamma a)\eta(\varphi) \right]. \quad (36)$$

Our goal is to determine acceleration generated by the electromagnetic field of the δ -form, which means that the four-potential A_μ is the Heaviside step function (18). To achieve this goal, let us define current density (Berestetzki *et al.*, 1989) as follows:

$$j^\mu = \bar{\Psi}_p \gamma^\mu \Psi_p, \quad (37)$$

where $\bar{\Psi}$ is defined as the transposition of (33), or

$$\bar{\Psi}_p = \frac{\bar{u}}{\sqrt{2p_0}} \left[1 + \frac{e}{2(kp)} (\gamma A)(\gamma k) \right] e^{-iS}. \quad (38)$$

After insertion of Ψ_p and $\bar{\Psi}_p$ into the current density, we have with $A_\mu = a_\mu \eta(\varphi)$, $\eta^2 = \eta$,

$$j^\mu = \frac{1}{p_0} \left\{ p^\mu - ea^\mu + k^\mu \left(\frac{e(pa)}{(kp)} - \frac{e^2 a^2}{2(kp)} \right) \right\} \quad (39)$$

for $\eta > 0$, which is evidently related to Eq. (28).

The so-called kinetic momentum corresponding to j^μ is as follows (Berestetzki *et al.*, 1989):

$$\begin{aligned} J^\mu &= \Psi_p^* (p^\mu - eA^\mu) \Psi_p = \bar{\Psi}_p \gamma^0 (p^\mu - eA^\mu) \Psi_p \\ &= \left\{ p^\mu - eA^\mu + k^\mu \left(\frac{e(pa)}{(kp)} - \frac{e^2 A^2}{2(kp)} \right) \right\} + k^\mu \frac{ie}{8(kp)p_0} F_{\alpha\beta} (u^* \sigma^{\alpha\beta} u), \end{aligned} \quad (40)$$

where

$$\sigma^{\alpha\beta} = \frac{1}{2} (\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha). \quad (41)$$

Now, we express the four-potential by the step function. In this case the kinetic momentum contains the tensor $F_{\mu\nu}$ involving δ -function. It means that there is a singularity at point $\varphi = 0$. This singularity plays no role in the situation for $\varphi > 0$

because in this case the δ -function is zero. Then, the kinetic momentum is the same as j^μ .

5. EMISSION OF PHOTONS BY AN ELECTRON MOVING IN THE IMPULSIVE FIELD

We know that the matrix element M corresponding to the emission of photon by electron in the electromagnetic field is as follows (Berestetzki *et al.*, 1989):

$$M = -ie^2 \int d^4x \bar{\Psi}_{p'}(\gamma e'^*) \Psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}}, \quad (42)$$

where Ψ_p is the wave function of an electron before interaction with the laser pulse and $\Psi_{p'}$ is the wave function of electron after emission of photon with components $k'^\mu = (\omega', \mathbf{k}')$. The quantity e'^* is the four polarization vector of emitted photon.

We write the matrix element in the more standard form

$$M = g \int d^4x \bar{\Psi}_{p'} O \Psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}}, \quad (43)$$

where $O = \gamma e'^*$, $g = -ie^2$ in case of the electromagnetic interaction and

$$\bar{\Psi}_{p'} = \frac{\bar{u}}{\sqrt{2p'_0}} \bar{R}(p') e^{-iS(p')}. \quad (44)$$

Using the above definitions, we write the matrix element in the form

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0 2p_0}} \int d^4x \bar{u} p' \bar{R}(p') O R(p) u(p) e^{-iS(p') + iS(p)} e^{ik'x}. \quad (45)$$

The quantity $\bar{R}(p')$ follows immediately from Eq. (33), namely

$$\bar{R}(p') = \left[1 + \frac{e}{2kp'} (\gamma k)(\gamma a)\eta(\varphi) \right] = \left[1 + \frac{e}{2kp'} (\gamma a)(\gamma k)\eta(\varphi) \right]. \quad (46)$$

Using

$$-iS(p') + iS(p) = i(p' - p) + i(\alpha' - \alpha)\varphi, \quad (47)$$

where

$$\alpha = \left(e \frac{ap'}{kp} - \frac{e^2 a^2}{2kp} \right), \quad \alpha' = \left(e \frac{ap'}{kp'} - \frac{e^2 a^2}{2kp'} \right), \quad (48)$$

we get

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0 2p_0}} \int d^4x \bar{u} p' \bar{R}(p') O R(p) u(p) e^{i(p'-p)x} e^{i(\alpha'-\alpha)\varphi} e^{ik'x}. \quad (49)$$

With regard to the mathematical relation $\eta^2(\varphi) = \eta(\varphi)$, we can put

$$\bar{R}(p')OR(p) = A + B\eta(\varphi), \quad (50)$$

where

$$A = \gamma e'^* \quad (51)$$

and

$$\begin{aligned} B = & \frac{e}{2(kp)}(\gamma e'^*)(\gamma k)(\gamma a) + \frac{e}{2(kp')}(\gamma a)(\gamma k)(\gamma e'^*) \\ & + \frac{e^2}{4(kp)(kp')}(\gamma a)(\gamma k)(\gamma e'^*)(\gamma k)(\gamma a). \end{aligned} \quad (52)$$

The total probability of the emission of photons during the interaction of the laser pulse with electron is as follows:

$$W = \int \frac{1}{2} \sum_{\text{spin, polar.}} \frac{|M|^2 d^3 p' d^3 k'}{VT (2\pi)^6}. \quad (53)$$

It is evident that the total calculation is complex and involves many algebraic operations with γ -matrices and δ -functions. At this moment we restrict the calculations to the most simple approximation where we replace the term in brackets in Eq. (33) by unity and so we write instead of Eq. (33)

$$\Psi_p \sim \frac{u}{\sqrt{2p_0}} e^{iS}, \quad (54)$$

which is usually used in similar form for the nonrelativistic calculations as it is discussed by Kreinov *et al.* (1997). Then, in this simplified situation $\bar{R}OR$ reduces to $A = \gamma e'^*$ and

$$\begin{aligned} M = & \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0} 2p_0} \int d^4 x \bar{u}(\gamma e'^*) u e^{i(p'-p)x} e^{i(\alpha'-\alpha)\varphi} e^{ik'x} \\ = & \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0} 2p_0} \bar{u}(\gamma e'^*) u \delta^{(4)}(lk + p - p' - k'), \end{aligned} \quad (55)$$

where

$$l = \alpha - \alpha'. \quad (56)$$

One important step is the determination of W in the determination of trace, because according to the quantum electrodynamics of a spin states it is possible to show that (Berestetskii *et al.*, 1989)

$$\frac{1}{2} \sum_{\text{spin, polar.}} |L|^2 = \frac{1}{2} \text{Tr}\{(\gamma p' + m)A(\gamma p + m)\gamma^0 A^+ \gamma^0\}, L = \bar{\mu}' Au \quad (57)$$

In order to determine trace Tr of the combinations of γ -matrix, it is suitable to know some relations. For instance

$$\text{Tr}(a\gamma)(b\gamma) = 4ab, \quad \text{Tr}(a\gamma)(b\gamma)(c\gamma) = 0, \quad (58)$$

$$\text{Tr}(a\gamma)(b\gamma)(c\gamma)(d\gamma) = 4[(ab)(cd) - (ac)(bd) + (ad)(bc)]. \quad (59)$$

Then,

$$\text{Tr}[(\gamma p' + m)A(\gamma p + m)\bar{A}] = S_1 + S_2 + S_3 + S_4, \quad \bar{A} = \gamma^0 A^+ \gamma^0, \quad (60)$$

where (using relations (58) and (59) and $\gamma^{\bar{\mu}} = \gamma^\mu$ with $e'e'^* = -1$)

$$S_1 = \text{Tr}[\gamma p' A \gamma p \bar{A}] = 4[(p'e'^*)(pe') + (pp') + (p'e')(pe'^*)], \quad (61)$$

$$S_2 = \text{Tr}[m A \gamma p \bar{A}] = 0, \quad (62)$$

$$S_3 = \text{Tr}[m \gamma p' A \bar{A}] = 0, \quad (63)$$

$$S_4 = \text{Tr}[m^2 A \bar{A}] = 4m^2(e'e'^*) = -4m^2. \quad (64)$$

At this moment we can write probability of the process W in the form

$$\begin{aligned} W &= \int \frac{1}{2} \sum_{\text{spin. polar.}} \frac{|M|^2 d^3 p' d^3 k'}{VT (2\pi)^6} \\ &\sim \int \frac{d^3 p' d^3 k'}{(2\pi)^6} \frac{1}{2} (S_1 + S_2 + S_3 + S_4) \frac{1}{(2\pi)^2} \delta^{(4)}(lk + p - p' - k') \\ &= \int \frac{d^3 p' d^3 k'}{(2\pi)^6} \frac{1}{2} \delta^{(4)}(lk + p - p' + k') \\ &\quad \times 4\{(p'e'^*)(pe') + (pp') - m^2 + (p'e')(pe'^*)\} \end{aligned} \quad (65)$$

The presence of the δ -function in the last formula is expression of the conservation law $lk + p = k' + p'$, which we write in the form

$$lk + p - k' = p'. \quad (66)$$

If we introduce the angle Θ between \mathbf{k} and \mathbf{k}' , then with $|\mathbf{k}| = \omega$ and $|\mathbf{k}'| = \omega'$, we get from the squared equation (66) in the rest system of electron, where $p = (m, 0)$, the following equation:

$$l \frac{1}{\omega'} - \frac{1}{\omega} = \frac{l}{m} (1 - \cos \Theta), \quad l = \alpha - \alpha', \quad (67)$$

which is modification of the original equation for the Compton process,

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m} (1 - \cos \Theta). \quad (68)$$

We see that the substantial difference between single photon interaction and δ -pulse interaction is the factor $s = \alpha - \alpha'$.

We know that the last formula of the original Compton effect can be written in the form suitable for the experimental verification, namely

$$\Delta\lambda = 4\pi \frac{\hbar}{mc} \sin^2 \frac{\Theta}{2}, \quad (69)$$

which was used by Compton for the verification of the quantum nature of light (Rohlf, 1994).

We can express Eq. (67) in new form. From equation $lk + p = k' + p'$, we get the following after multiplying it by k in the rest frame of electron:

$$kp' = \omega m - \omega\omega'(1 - \cos \Theta). \quad (70)$$

Then, l in Eq. (67) is given by the formula ($a \equiv (v, \mathbf{w})$):

$$l = \frac{2evm - e^2a^2}{2\omega m} - \frac{2eap' - e^2a^2}{2\omega[m - \omega'(1 - \cos \Theta)]}. \quad (71)$$

Equation (67) can be experimentally verified by the similar methods which were used by Compton for the verification of his formula. However, it seems that the interaction of the photonic pulse substantially differs from the interaction of a single photon with electron.

The equation $lk + p = k' + p'$ is the symbolic expression of the nonlinear Compton effect in which several photons are absorbed at a single point, but only single high-energy photon is emitted. The second process, where electron scatters twice or more as it traverses the laser focus, is not considered here. The nonlinear Compton process was experimentally confirmed, for instance, by Bulla *et al.* (1996).

6. DISCUSSION

In this paper we have presented the classical derivation of law of motion of a charged particle accelerated by δ -form mechanical impulsive force in case of the free particle and for the damped harmonic oscillator. Then, we found solution of the Lorentz equation for motion of a charged particle accelerated by the electromagnetic pulse. From the quantum theory based on the Volkov solution of the Dirac equation, we determined the current density and kinetic momentum of an electron accelerated by the laser pulse. The total probability of emission of photons during the interaction of the laser pulse with electron was also derived. It involves the relation between initial momenta of particle and photon and final ones.

The present paper is continuation of the author's discussion on laser acceleration (Pardy, 1998, 2001), where the Compton model of laser acceleration was proposed.

The δ -form laser pulses are considered here as an idealization of the experimental situation in laser physics. Nevertheless, it was demonstrated theoretically that at present time the zeptosecond and subzeptosecond laser pulses of duration

$10^{-21} - 10^{-22}$ s can be realized by the petawatt lasers (Kaplan and Shkolnikov, 2002). It means that the generation of the ultrashort laser pulses is of keen interest in development of laser physics.

New experiments can be realized and new measurements performed by means of the laser pulses, giving new results and discoveries. So, it is obvious that the interaction of particles with the laser pulses can form, in the near future, the integral part of the laser and particle physics in such laboratories as ESRF, CERN, DESY, and SLAC.

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